Одномерные теч-я газа

Рассм. одномерное, плоское, нестац-ое теч-е газа:

1) одномерное: $\frac{\partial \cdot}{\partial x_2} = \frac{\partial \cdot}{\partial x_3} = 0;\ \frac{\partial \cdot}{\partial x_1} = \frac{\partial}{\partial x} \neq 0$

2) плоское: $v(v_1, 0, 0)$

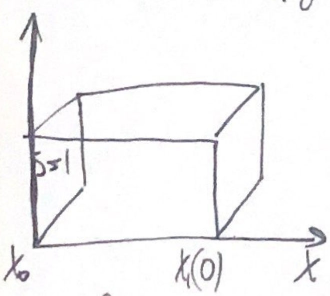
3) нестац-ое: $\frac{\partial}{\partial t} \neq 0$

$$\frac{\partial \rho}{\partial t} + \operatorname{div}(\rho v) = 0 \Rightarrow \frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho v) = 0;\ \frac{\partial v}{\partial t} + (v, \nabla) v + \frac{1}{\rho}\operatorname{grad} p = 0 \Rightarrow$$

$$\Rightarrow \frac{\partial v}{\partial t} + v\frac{\partial v}{\partial x} + \frac{1}{\rho}\frac{\partial p}{\partial x} = 0;\ \frac{\partial}{\partial t}\left(\varepsilon + \frac{v^2}{2}\right) + (v, \nabla)\left(\varepsilon + \frac{v^2}{2}\right) + \frac{1}{\rho}\operatorname{div}(p v) = 0$$

$$\Rightarrow \frac{\partial}{\partial t}\left(\varepsilon + \frac{v^2}{2}\right) + v\frac{\partial}{\partial x}\left(\varepsilon + \frac{v^2}{2}\right) + \frac{1}{\rho}\frac{\partial}{\partial x}(p v) = 0$$

Массовые коорд-ты Лагранжа



$S=1$ | x_0 - левая стенка неподв-на. внутри парал. газ,
$x_1(t)$ | вне вакуум. $t_0 = 0$ стенку (правую) мгновенно убирают $\Rightarrow$ истечение газа в вакуум. в силу од-ти теч-я перемещение газа только вдоль оси x.

газ в любой момент времени $t > 0$ имеет форму параллелепипеда.

масса газа сохр-ся

$$s = \int_{x_0}^{x_1(0)} \rho \underbrace{S}_{\text{площадь}} dx = \int_{x_0}^{x_1(0)} \rho\, dx = M \text{ - вся масса газа};\quad s = \int_{x_0}^{x_1(t)} \rho\, dx = \int_{x_0}^{x} \rho(y, t)\, dy$$

масса газа от x_0 до $x_1(0)$

s (масса газа) связана с x

$$\underset{(\text{эйлер.})}{f(t_э, x)} \to \underset{(\text{лагр.})}{f(t_л, s)};\ \frac{\partial f}{\partial t_э} = \frac{\partial f}{\partial t_л}\cdot\frac{\partial t_л}{\partial t_э} + \frac{\partial f}{\partial s}\cdot\frac{\partial s}{\partial t_э};\ \frac{\partial f}{\partial x} = \frac{\partial f}{\partial t_л}\cdot\frac{\partial t_л}{\partial x} + \frac{\partial f}{\partial s}\frac{\partial s}{\partial x}$$

в силу $t_л = t_э \Rightarrow \frac{\partial t_л}{\partial t_э} = 1,\ \frac{\partial t_л}{\partial x} = 0,\ t_э = t;\ s = \int_{x_0}^{x} \rho(t, y)\, dy;\ \frac{\partial s}{\partial x} = \frac{\partial}{\partial x}\int_{x_0}^{x} \rho(t, y)\, dy = \rho$

$$\frac{\partial s}{\partial t} = \int_{x_0}^{x} \frac{\partial}{\partial t}\rho(t, y)\, dy = -\int_{x_0}^{x} \frac{\partial}{\partial y}\big(\rho(t, y)\cdot v(t, y)\big)\, dy \ominus \quad \Bigg| \quad \frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho v) = 0 \Rightarrow \frac{\partial \rho}{\partial t} = -\frac{\partial}{\partial x}(\rho v)$$

⑤ $-\rho v\Big|_{x_0}^{x} = -\rho v\Big|_{x}$; x_0 - неподв-ная $\Rightarrow v(x_0)=0$; $\dfrac{\partial f}{\partial t} = \dfrac{\partial f}{\partial t_\Lambda} - \rho v \dfrac{\partial f}{\partial s}$, ← масса в коорд. лагр

$\dfrac{\partial f}{\partial x} = \rho \dfrac{\partial f}{\partial s}$; $\dfrac{\partial \rho}{\partial t} + \dfrac{\partial}{\partial x}(\rho v) = 0$; $\dfrac{\partial \rho}{\partial t_\Lambda} - \rho v \dfrac{\partial \rho}{\partial s} + \rho \dfrac{\partial}{\partial s}(\rho v) = 0$;

$\dfrac{\partial \rho}{\partial t_\Lambda} - \cancel{\rho v \dfrac{\partial \rho}{\partial s}} + \cancel{\rho v \dfrac{\partial \rho}{\partial s}} + \rho^2 \dfrac{\partial}{\partial s} v = 0$; $-\dfrac{1}{\rho^2}\dfrac{\partial \rho}{\partial t_\Lambda} - \dfrac{\partial v}{\partial s} = 0$;

$\dfrac{\partial}{\partial t_\Lambda}\left(\dfrac{1}{\rho}\right) = \dfrac{\partial v}{\partial s}$; $\dfrac{d}{dt}\left(\dfrac{1}{\rho}\right) = \dfrac{\partial v}{\partial s}$;

$\dfrac{\partial v}{\partial t} + v\dfrac{\partial v}{\partial x} + \dfrac{1}{\rho}\dfrac{\partial p}{\partial x} = 0$; $\underbrace{\dfrac{\partial v}{\partial t_\Lambda} - \rho v \dfrac{\partial v}{\partial s}} + v\rho \dfrac{\partial v}{\partial s} + \dfrac{1}{\rho}\rho \dfrac{\partial p}{\partial s} = 0$

$\dfrac{\partial v}{\partial t_\Lambda} = -\dfrac{\partial p}{\partial s}$; $\dfrac{dv}{dt} = -\dfrac{\partial p}{\partial s}$;

$$\frac{\partial}{\partial t}\left(\varepsilon + \frac{v^2}{2}\right) + v\frac{\partial}{\partial x}\left(\varepsilon + \frac{v^2}{2}\right) + \frac{1}{\rho}\frac{\partial}{\partial x}(pv) = 0$$

$$\frac{d}{dt}\left(\varepsilon + \frac{v^2}{2}\right) - \rho v\frac{\partial}{\partial s}\left(\varepsilon + \frac{v^2}{2}\right) + v\rho\frac{\partial\left(\varepsilon + \frac{v^2}{2}\right)}{\partial s} + \frac{1}{\rho}\rho\frac{\partial (pv)}{\partial s} = 0$$

$$\frac{d}{dt}\left(\varepsilon + \frac{v^2}{2}\right) = -\frac{\partial}{\partial s}(pv)$$

$p(\rho, T)$, $\varepsilon(\rho, T)$

$\dfrac{dx}{dt} = v$